3.4 Exercises

Vocabulary Check

Fill in the blanks.

1. To _____ an equation in x means to find all values of x for which the equation is true.
2. To solve exponential and logarithmic equations, you can use the following one-to-one and inverse properties.
   (a) $a^x = a^y$ if and only if _____. (b) $\log_a x = \log_a y$ if and only if _____.
   (c) $a^{\log_a x} =$ _____ (d) $\log_a a^x =$ _____
3. An _____ solution does not satisfy the original equation.

In Exercises 1–8, determine whether each x-value is a solution of the equation.

1. $4^{2x-7} = 64$
   (a) $x = 5$
   (b) $x = 2$
2. $2^{3x+1} = 32$
   (a) $x = -1$
   (b) $x = 2$
3. $3e^{x+2} = 75$
   (a) $x = -2 + e^{25}$
   (b) $x = -2 + \ln 25$
   (c) $x \approx 1.2189$
4. $4e^{x-1} = 60$
   (a) $x = 1 + \ln 15$
   (b) $x \approx 3.7081$
   (c) $x = \ln 16$
5. $\log_4(3x) = 3$
   (a) $x \approx 21.3560$
   (b) $x = -4$
   (c) $x = \frac{64}{3}$
6. $\log_6(\frac{5}{3}x) = 2$
   (a) $x \approx 20.2882$
   (b) $x = \frac{108}{5}$
   (c) $x = 7.2$
7. $\ln(x - 1) = 3.8$
   (a) $x = 1 + e^{3.8}$
   (b) $x \approx 45.7012$
   (c) $x = 1 + \ln 3.8$
8. $\ln(2 + x) = 2.5$
   (a) $x = e^{2.5} - 2$
   (b) $x \approx \frac{4073}{400}$
   (c) $x = \frac{1}{2}$

In Exercises 9–16, use a graphing utility to graph f and g in the same viewing window. Approximate the point of intersection of the graphs of f and g. Then solve the equation $f(x) = g(x)$ algebraically.

9. $f(x) = 2^x$, $g(x) = 8$
10. $f(x) = 27^x$, $g(x) = 9$
11. $f(x) = 5^{x-2} - 15$, $g(x) = 10$
12. $f(x) = 2^{-x+1} - 3$, $g(x) = 13$
13. $f(x) = 4 \log_3 x$, $g(x) = 20$
14. $f(x) = 3 \log_5 x$, $g(x) = 6$
15. $f(x) = \ln e^{x+1}$, $g(x) = 2x + 5$
16. $f(x) = \ln e^{x-2}$, $g(x) = 3x + 2$

In Exercises 17–36, solve for x.

17. $4^x = 16$
18. $3^x = 243$
19. $5^x = \frac{1}{625}$
20. $7^x = \frac{1}{49}$
21. $(\frac{1}{8})^x = 64$
22. $(\frac{1}{2})^x = 32$
23. $(\frac{2}{3})^x = \frac{81}{16}$
24. $(\frac{3}{4})^x = \frac{27}{64}$
25. $e^x = 4$
26. $e^x = 0$
27. $\ln x - \ln 5 = 0$
28. $\ln x - \ln 2 = 0$
29. $\ln x = -7$
30. $\ln x = -1$
31. $\log_x 625 = 4$
32. $\log_x 25 = 2$
33. $\log_{10} x = -1$
34. $\log_{10} x = -\frac{1}{2}$
35. $\ln(2x - 1) = 5$
36. $\ln(3x + 5) = 8$

In Exercises 37–42, simplify the expression.

37. $\ln e^{x^2}$
38. $\ln e^{2x-1}$
39. $e^{\ln(5x+2)}$
40. $-1 + \ln e^{2x}$
41. $e^{\ln x^2}$
42. $-8 + e^{\ln x^3}$

In Exercises 43–60, solve the exponential equation algebraically. Round your result to three decimal places. Use a graphing utility to verify your answer.

43. $8^{3x} = 360$
44. $6^{5x} = 3000$
45. $2e^{5x} = 18$
46. $4e^{2x} = 40$
47. $500e^{-x} = 300$
48. $1000e^{-4x} = 75$
49. $7 - 2e^x = 5$
50. $-14 + 3e^x = 11$
51. $5^{-t/2} = 0.20$
52. $4^{-3t} = 0.10$
53. $2^{3-x} = 565$
54. $8^{-2-x} = 431$
55. $e^{2x} - 4e^x - 5 = 0$
56. $e^{2x} - 5e^x + 6 = 0$
57. $\frac{400}{1 + e^{-x}} = 350$
58. $\frac{525}{1 + e^{-x}} = 275$

59. $\left(1 + \frac{0.10}{12}\right)^{12t} = 2$ 60. $\left(16 + \frac{0.878}{26}\right)^{3t} = 30$

In Exercises 61–64, complete the table to find an interval containing the solution of the equation. Then use a graphing utility to graph both sides of the equation to estimate the solution. Round your result to three decimal places.

61. $e^{3x} = 12$

x	0.6	0.7	0.8	0.9	1.0
e^{3x}					

62. $e^{2x} = 50$

x	1.6	1.7	1.8	1.9	2.0
e^{2x}					

63. $20(100 - e^{x/2}) = 500$

x	5	6	7	8	9
$20(100 - e^{x/2})$					

64. $\frac{400}{1 + e^{-x}} = 350$

x	0	1	2	3	4
$\frac{400}{1 + e^{-x}}$					

In Exercises 65–68, use the *zero* or *root* feature or the *zoom* and *trace* features of a graphing utility to approximate the solution of the exponential equation accurate to three decimal places.

65. $\left(1 + \frac{0.065}{365}\right)^{365t} = 4$ 66. $\left(4 - \frac{2.471}{40}\right)^{9t} = 21$

67. $\frac{3000}{2 + e^{2x}} = 2$ 68. $\frac{119}{e^{6x} - 14} = 7$

In Exercises 69–72, use a graphing utility to graph the function and approximate its zero accurate to three decimal places.

69. $g(x) = 6e^{1-x} - 25$ 70. $f(x) = 3e^{3x/2} - 962$

71. $g(t) = e^{0.09t} - 3$ 72. $h(t) = e^{0.125t} - 8$

In Exercises 73–92, solve the logarithmic equation algebraically. Round the result to three decimal places. Verify your answer using a graphing utility.

73. $\ln x = -3$ 74. $\ln x = -2$

75. $\ln 4x = 2.1$ 76. $\ln 2x = 1.5$

77. $-2 + 2 \ln 3x = 17$ 78. $3 + 2 \ln x = 10$

79. $\log_{10}(z - 3) = 2$ 80. $\log_{10} x^2 = 6$

81. $7 \log_4(0.6x) = 12$ 82. $4 \log_{10}(x - 6) = 11$

83. $\ln \sqrt{x + 2} = 1$ 84. $\ln \sqrt{x - 8} = 5$

85. $\ln(x + 1)^2 = 2$ 86. $\ln(x^2 + 1) = 8$

87. $\log_4 x - \log_4(x - 1) = \frac{1}{2}$

88. $\log_3 x + \log_3(x - 8) = 2$

89. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

90. $\ln(x + 1) - \ln(x - 2) = \ln x$

91. $\log_{10} 8x - \log_{10}(1 + \sqrt{x}) = 2$

92. $\log_{10} 4x - \log_{10}(12 + \sqrt{x}) = 2$

In Exercises 93–96, complete the table to find an interval containing the solution of the equation. Then use a graphing utility to graph both sides of the equation to estimate the solution. Round your result to three decimal places.

93. $\ln 2x = 2.4$

x	2	3	4	5	6
$\ln 2x$					

94. $3 \ln 5x = 10$

x	4	5	6	7	8
$3 \ln 5x$					

95. $6 \log_3(0.5x) = 11$

x	12	13	14	15	16
$6 \log_3(0.5x)$					

96. $5 \log_{10}(x - 2) = 11$

x	150	155	160	165	170
$5 \log_{10}(x - 2)$					

In Exercises 97–102, use the *zero* or *root* feature or the *zoom* and *trace* features of a graphing utility to approximate the solution of the logarithmic equation accurate to three decimal places.

97. $\log_{10} x = x^3 - 3$

98. $\log_{10} x^2 = 4$

99. $\log_3 x + \log_3(x - 3) = 1$

100. $\log_2 x + \log_2(x + 5) = 4$

101. $\ln(x - 3) + \ln(x + 3) = 1$

102. $\ln x + \ln(x^2 + 4) = 10$

In Exercises 103–108, use a graphing utility to approximate the point of intersection of the graphs. Round your result to three decimal places.

103. $y_1 = 7$, $y_2 = 2^{x-1} - 5$

104. $y_1 = 4$, $y_2 = 3^{x+1} - 2$

105. $y_1 = 80$, $y_2 = 4e^{-0.2x}$

106. $y_1 = 500$, $y_2 = 1500e^{-x/2}$

107. $y_1 = 3.25$, $y_2 = \frac{1}{2}\ln(x + 2)$

108. $y_1 = 1.05$, $y_2 = \ln\sqrt{x - 2}$

Compound Interest **In Exercises 109 and 110, find the time required for a $1000 investment to (a) double at interest rate r, compounded continuously, and (b) triple at interest rate r, compounded continuously.**

109. $r = 0.085$

110. $r = 0.12$

111. ***Demand*** The demand equation for a camera is given by

$$p = 500 - 0.5(e^{0.004x}).$$

Find the demand x for a price of (a) $p = \$350$ and (b) $p = \$300$.

112. ***Demand*** The demand equation for a hand-held electronic organizer is given by

$$p = 5000\left(1 - \frac{4}{4 + e^{-0.002x}}\right).$$

Find the demand x for a price of (a) $p = \$600$ and (b) $p = \$400$.

113. ***Forestry*** The number of trees per acre N of a certain species is approximated by the model

$$N = 68(10^{-0.04x}), \quad 5 \le x \le 40$$

where x is the average diameter of the trees (in inches) three feet above the ground. Use the model to approximate the average diameter of the trees in a test plot for which $N = 21$.

114. ***Forestry*** The yield V (in millions of cubic feet per acre) for a forest at age t years is given by

$$V = 6.7e^{-48.1/t}.$$

(a) Use a graphing utility to graph the function.

(b) Determine the horizontal asymptote of the function. Interpret its meaning in the context of the problem.

(c) Find the time necessary to obtain a yield of 1.3 million cubic feet.

115. ***Average Heights*** The percent m of American males between the ages of 18 and 24 who are no more than x inches tall is modeled by

$$m(x) = \frac{100}{1 + e^{-0.6114(x-69.71)}}$$

and the percent f of American females between the ages of 18 and 24 who are no more than x inches tall is modeled by

$$f(x) = \frac{100}{1 + e^{-0.66607(x-64.51)}}.$$

(Source: U.S. National Center for Health Statistics)

(a) Use a graphing utility to graph the two functions in the same viewing window.

(b) Use the graphs in part (a) to determine the horizontal asymptotes of the functions. Interpret their meaning in the context of the problem.

(c) What is the average height for each sex?

116. ***Human Memory Model*** In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.83}{1 + e^{-0.2n}}.$$

(a) Use a graphing utility to graph the function.

(b) Use the graph in part (a) to determine any horizontal asymptotes of the function. Interpret the meaning of the upper asymptote in the context of the problem.

(c) After how many trials will 60% of the responses be correct?