Lesson 25: Adding and Subtracting Rational Expressions

* need LCD *
(lowest common denominator)

Classwork

Exercises 1–4

1. Calculate the following sum: $\frac{3}{10}+\frac{6}{10}$. $= \frac{3+6}{10} = \boxed{\frac{9}{10}}$

2. $\frac{3}{20}-\frac{4}{15}$

LCD: 20, 40, 60, 80, 100
15, 30, 45, 60
So LCD is 60

$\frac{9}{60} - \frac{16}{60} = \frac{9-16}{60} = \boxed{\frac{-7}{60}}$

3. $\frac{\pi}{4}+\frac{\sqrt{2}}{5}$

4, 8, 12, 16, 20
5, 10, 15, 20
So LCD is 20

$\frac{5\pi}{20} + \frac{4\sqrt{2}}{20} = \boxed{\frac{5\pi+4\sqrt{2}}{20}}$

4. $\frac{a}{m}+\frac{b}{2m}-\frac{c}{m}$

LCD: m
$2\cdot m$
m
So LCD is $2m$

$\frac{2a}{2m} + \frac{b}{2m} - \frac{2c}{2m} = \boxed{\frac{2a+b-2c}{2m}}$

Example 1

Perform the indicated operations below and simplify.

a. $\frac{a+b}{4}+\frac{2a-b}{5}$

LCD: 4, 8, 12, 16, 20 so LCD is 20
5, 10, 15, 20

$$\frac{5(a+b)}{20}+\frac{4(2a-b)}{20}$$

$$=\frac{5a+5b}{20}+\frac{8a-4b}{20}=\frac{5a+5b+8a-4b}{20}=\boxed{\frac{13a+b}{20}}$$

b. $\frac{4}{3x}-\frac{3}{5x^2}$

15
$3\cdot x$
$5\cdot x\cdot x$

LCD: $15x\cdot x=15x^2$

$$\frac{4(5x)}{15x^2}-\frac{3(3)}{15x^2}$$

$$=\frac{20x}{15x^2}-\frac{9}{15x^2}=\boxed{\frac{20x-9}{15x^2}}$$

c. $\frac{3}{2x^2+2x}+\frac{5}{x^2-3x-4}$

$2x^2+2x \rightarrow 2x(x+1)$
$x^2-3x-4 \rightarrow (x-4)(x+1)$

LCD: $2x(x-4)(x+1)$

$$\frac{3(x-4)}{2x(x-4)(x+1)}+\frac{5(2x)}{2x(x-4)(x+1)}$$

$$\frac{3x-12}{2x(x-4)(x+1)}+\frac{10x}{2x(x-4)(x+1)}=\frac{3x-12+10x}{2x(x-4)(x+1)}=\boxed{\frac{13x-12}{2x(x-4)(x+1)}}$$

Exercises 5–8

Perform the indicated operations for each problem below.

5. $\frac{5}{x-2}+\frac{3x}{4x-8}$

$x-2$

$4x-8 \rightarrow 4(x-2)$ LCD: $4(x-2)$

$\frac{20}{4(x-2)}+\frac{3x}{4(x-2)}=\boxed{\frac{20+3x}{4(x-2)}}$

"−1 trick"

6. $\frac{7m}{m-3}+\frac{5m}{3-m}$

$m-3$

$3-m \rightarrow -1(m-3)$ LCD: $-1(m-3)$

$\frac{-7m}{-1(m-3)}+\frac{5m}{-1(m-3)}=\frac{-7m+5m}{-1(m-3)}=\boxed{\frac{-2m}{-1(m-3)}}$

7. $\frac{b^2}{b^2-2bc+c^2}-\frac{b}{b-c}$

$b^2-2bc+c^2 \rightarrow (b-c)(b-c)$ LCD: $(b-c)(b-c)$

$b-c$

$\frac{b^2}{(b-c)(b-c)}-\frac{b(b-c)}{(b-c)(b-c)}$

$\frac{b^2}{(b-c)(b-c)}-\frac{b^2-bc}{(b-c)(b-c)}=\frac{b^2-(b^2-bc)}{(b-c)(b-c)}=\boxed{\frac{bc}{(b-c)(b-c)}}$

8. $\frac{x}{x^2-1}-\frac{2x}{x^2+x-2}$

$x^2-1 \rightarrow (x-1)(x+1)$

$x^2+x-2 \rightarrow (x+2)(x-1)$ LCD: $(x-1)(x+1)(x+2)$

$\frac{x(x+2)}{(x-1)(x+1)(x+2)}-\frac{2x(x+1)}{(x-1)(x+1)(x+2)}=\boxed{\frac{-x^2}{(x+1)(x-1)(x+2)}}$

$\frac{x^2+2x}{(x-1)(x+1)(x+2)}-\frac{2x^2+2x}{(x-1)(x+1)(x+2)}=\frac{x^2+2x-(2x^2+2x)}{(x-1)(x+1)(x+2)}$

Example 2

Simplify the following expression.

Follow 1 → 3

$$\frac{\left(\frac{b^2+b-1}{2b-1}-1\right)}{\left(4-\frac{8}{(b+1)}\right)} \rightarrow \underbrace{\left(\frac{b^2+b-1}{2b-1}-1\right)}_{\text{make 1 fraction}} \div \underbrace{\left(4-\frac{8}{b+1}\right)}_{\text{make 1 fraction}}$$

1

$$\frac{b^2+b-1}{2b-1}-1$$

$$\frac{b^2+b-1}{2b-1}-\frac{2b-1}{2b-1}$$

$$\frac{b^2+b-1-(2b-1)}{2b-1}$$

$$\frac{b^2+b\cancel{-1}-2b\cancel{+1}}{2b-1}$$

$$\frac{b^2-b}{2b-1}$$

3

$$\frac{b^2-b}{2b-1}\cdot\frac{b+1}{4b-4}$$

$$\frac{b\cancel{(b-1)}\cdot b+1}{2b-1\cdot 4\cancel{(b-1)}}$$

$$=\boxed{\frac{b(b+1)}{4(2b-1)}}$$

2

$$4-\frac{8}{b+1}$$

$$\frac{4}{1}-\frac{8}{b+1}$$

$$\frac{4(b+1)}{b+1}-\frac{8}{b+1}$$

$$\frac{4b+4}{b+1}-\frac{8}{b+1}$$

$$\frac{4b+4-8}{b+1}=\frac{4b-4}{b+1}$$

Lesson Summary

In this lesson, we extended addition and subtraction of rational numbers to addition and subtraction of rational expressions. The process for adding or subtracting rational expressions can be summarized as follows:

- Find a common multiple of the denominators to use as a common denominator.
- Find equivalent rational expressions for each expression using the common denominator.
- Add or subtract the numerators as indicated and simplify if needed.

Problem Set

1. Write each sum or difference as a single rational expression.

   a. $\frac{7}{8} - \frac{\sqrt{3}}{5}$

   b. $\frac{\sqrt{5}}{10} + \frac{\sqrt{2}}{6} + 2$

   c. $\frac{4}{x} + \frac{3}{2x}$

2. Write as a single rational expression.

   a. $\frac{1}{x} - \frac{1}{x-1}$

   b. $\frac{3x}{2y} - \frac{5x}{6y} + \frac{x}{3y}$

   c. $\frac{a-b}{a^2} + \frac{1}{a}$

   d. $\frac{1}{p-2} - \frac{1}{p+2}$

   e. $\frac{1}{p-2} + \frac{1}{2-p}$

   f. $\frac{1}{b+1} - \frac{b}{1+b}$

   g. $1 - \frac{1}{1+p}$

   h. $\frac{p+q}{p-q} - 2$

   i. $\frac{r}{s-r} + \frac{s}{r+s}$

   j. $\frac{3}{x-4} + \frac{2}{4-x}$

   k. $\frac{3n}{n-2} + \frac{3}{2-n}$

   l. $\frac{8x}{3y-2x} + \frac{12y}{2x-3y}$

   m. $\frac{1}{2m-4n} - \frac{1}{2m+4n} - \frac{m}{m^2-4n^2}$

   n. $\frac{1}{(2a-b)(a-c)} + \frac{1}{(b-c)(b-2a)}$

   o. $\frac{b^2+1}{b^2-4} + \frac{1}{b+2} + \frac{1}{b-2}$

3. Write each rational expression as an equivalent rational expression in lowest terms.

   a. $\dfrac{\frac{1}{a} - \frac{1}{2a}}{\frac{4}{a}}$

   b. $\dfrac{\frac{5x}{2} + 1}{\frac{5x}{4} - \frac{1}{5x}}$

   c. $\dfrac{1 + \frac{4x+3}{x^2+1}}{1 - \frac{x+7}{x^2+1}}$

Extension:

4. Suppose that $x \neq 0$ and $y \neq 0$. We know from our work in this section that $\frac{1}{x} \cdot \frac{1}{y}$ is equivalent to $\frac{1}{xy}$. Is it also true that $\frac{1}{x} + \frac{1}{y}$ is equivalent to $\frac{1}{x+y}$? Provide evidence to support your answer.

5. Suppose that $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$. Show that the value of $x^2 + y^2$ does not depend on the value of t.

6. Show that for any real numbers a and b, and any integers x and y so that $x \neq 0$, $y \neq 0$, $x \neq y$, and $x \neq -y$,

$$\left(\frac{y}{x} - \frac{x}{y}\right)\left(\frac{ax+by}{x+y} - \frac{ax-by}{x-y}\right) = 2(a-b).$$

7. Suppose that n is a positive integer.
   a. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)$.
   b. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)$.
   c. Rewrite the product in the form $\frac{P}{Q}$ for polynomials P and Q: $\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n+1}\right)\left(1+\frac{1}{n+2}\right)\left(1+\frac{1}{n+3}\right)$.
   d. If this pattern continues, what is the product of n of these factors?